are substituted in the following local energy flux expressions:

$$(-J_q/L_\lambda)_{y=0} = (PU_\infty/\gamma)(T_0-T_\infty)[1.5d'_1(3/20\Delta - 31/420\Delta^3)+d'_2(-3/8+3/40\Delta^2-3/280\Delta^4) + (nd_2/x)(-3/8+0.25/\Delta-3/40\Delta^2 + 1/280\Delta^4)+0.5(d_2/x)(-0.25+0.2/\Delta^2 - 17/210\Delta^4)] \quad (P \leqslant 1) \qquad \text{(A4)}$$

$$= (PU_\infty/\gamma)(T_0-T_\infty)[1.5d'_1(0.2\Delta^2-\Delta^3/6 +3\Delta^4/70)+d'_2(-0.6\Delta+3\Delta^2/8-3\Delta^3/35) +(nd_2/x)(-0.3\Delta+\Delta^2/8-3\Delta^3/140) +0.5(d_2/x)(-0.2\Delta+\Delta^2/12 -\Delta^3/70)] \quad (P \geqslant 1) \qquad \text{(A5)}$$

and the local heat transfer is computed.

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A note on the series solutions of momentum and energy equations for heat transfer from a semi-infinite plate

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1. INTRODUCTION

THE LAMINAR flow situation that is our concern here is zero pressure gradient and unity Prandtl number fluid flow over a semi-infinite plate of uniform temperature. For the flow under consideration, Meksyn [1] presented series solutions of the governing momentum and energy equations; the relevant equations and boundary conditions are

$$f'''+ff'' = 0 \qquad (1)$$

$$\theta''+f\theta' = 0 \qquad (2)$$

$$f(0) = f'(0) = 0; \quad f'(\infty) \to 1 \qquad (3)$$

$$\theta(0) = 1; \quad \theta(\infty) \to 0. \qquad (4)$$

Here f and θ denote the non-dimensional stream-function and temperature, respectively; primes denote derivatives with respect to y. In brief, Meksyn uses the Blasius series

$$f = \sum_{N=2}^{\infty} A_N y^N/N! = a(y^2/2!)-a^2(y^5/5!) +11a^3(y^8/8!)-375a^4(y^{11}/11!)+27\,897a^5(y^{14}/14!)\ldots \qquad (5)$$

where A_N are the coefficients and $f''(0) = a$, to integrate equations (1) and (2). The end result is that the temperature distribution is a combination of two series: one being in terms of the incomplete gamma function and the other for $\theta'(0)$. The series for $\theta'(0)$ is given by

$$\theta'(0) = -0.478/[1+1/45-1/405\ldots]. \qquad (6)$$

For the flow under consideration, the Reynolds analogy suggests that skin-friction is a direct measure of wall heat transfer rate [2]. However, relation (6) does not lead to this explicit relation. It appears that only the functional analysis of the Reynolds analogy between momentum and heat transfer can provide such an explicit relation. Also, it appears (to the author's knowledge) that no attempt was made in the past to arrive at this explicit relation using series solutions of equations (1)–(4). The aim of this note is to show that the series solutions of equations (1)–(4) can be used to develop this explicit relation between the skin-friction and wall heat transfer rate.

2. ANALYSIS

For the sake of convenience, we convert equations (1)–(4) into initial value problems using the following transformations and initial conditions:

$$Y = y/s, \quad F = sf, \quad a = 1/s^3, \quad (d\theta/dy)_{y=0} = c \text{ (say)}. \qquad (7)$$

The initial value problems to be solved are

$$F'''+FF'' = 0 \qquad (8)$$

$$\theta''+F\theta' = 0 \qquad (9)$$

$$F(0) = F'(0) = 0, \quad F''(0) = 1 \qquad (10)$$

$$\theta(0) = 1, \quad \theta'(0) = cs = b \text{ (say)}. \qquad (11)$$

Here (and in what follows) primes denote derivatives with respect to Y. The parameters s and b are to be estimated satisfying the conditions that $F'(Y) \to s^2$ and $\theta(Y) \to 0$ as $Y \to \infty$.

Using Maclaurin's series expansion one obtains from equations (8)–(11)

$$F = (Y^2/2!)-(Y^5/5!)+11(Y^8/8!)-375(Y^{11}/11!) +27\,897(Y^{14}/14!)-\cdots+R_M \qquad (12)$$

$$\theta = 1+bY-b(Y^4/4!)+11b(Y^7/7!)-375b(Y^{10}/10!) +27\,897b(Y^{13}/13!)\cdots+R_P. \qquad (13)$$

Here R_M and R_P denote the remainders. Comparing series (12) and (5), we note that they differ by the scaling factors introduced in equations (7).

Recently, Torok and Advani [3] have shown that a series solution to a non-linear initial value problem can be obtained via infinitesimal generators. Expressing equations (8) and (10) as three first-order differential equations (for the sake of brevity, details are not given here), one obtains series (12). In this case, this series represents a continuous group of transformations parameterized by Y. Given a point on a trajectory, which is an invariant curve of the transformation group [3], a point is mapped onto another along the trajectory as Y advances. This possibly implies that series (12) is not only true for small Y [2, 4], but for all Y.

However, series (12) does not appear to converge so easily in the sense that F' (obtained from series (12)) does not attain its asymptotic values as $Y \to \infty$. Shank's transformation [5] applied to only five terms of the series appears to accelerate the convergence rate; e.g. from Table 1 (this table contains the results for $Y = 3$ and 6) we see that at $Y = 6$, repeated use of this transformation drastically reduces the F' value from 52 826 ($= S_5$) to 2.07! Noting the observation of Van Dyke [6] that at least 15 terms are required to obtain the

NOMENCLATURE

a	value of the non-dimensional wall shear
A_N	coefficients, series (5)
b	constant, relation (11)
c	value of the non-dimensional wall heat transfer rate
e	Skank's transformation value
f	non-dimensional stream function
F	scaled value of f, relation (7)
m	number of terms considered in the Shank's transformation
N	number of terms, relation (5)
R_M, R_P	remainders of Maclurin's series, relations (12) and (13)
s	constant, relation (7)
s_m	partial sum of the series
y	non-dimensional coordinate
Y	scaled value of y.

Greek symbol

θ	non-dimensional temperature.

Superscript

′	derivative.

Table 1. Results of Shank's transformation [5] applied to the series for F' (obtained from series (12)) for large Y

Y	m	s_m	$e_1(s_m)$	e_1^2
3	1	3.0		
	2	−0.375	1.6	
	3	4.39	1.72	1.658
	4	−1.7	1.58	
	5	5.44		
6	1	6.0		
	2	−48.0	1.61	
	3	563.0	6.42	2.07
	4	−5685.0	−39.8	
	5	52 826.0		

Here m and s_m denote the number of terms and partial sum, respectively; $e_1(s_m) = (s_{m+1}s_{m-1}-s_m^2)\,(s_{m+1}+s_{m-1}-2s_m)^{-1}$; $e_1^2 = e_1(e_1(s_m))$.

correct radius of convergence of series (5), the conclusion that becomes obvious at this stage is that such a transformation applied to a large number of terms will ensure that $F'(Y) \to s^2$ as $Y \to \infty$. This in turn will enable us to estimate s^2 accurately; however, no such attempt was made as this is not the aim of this note. We may note that the numerical solution of equations (8) and (11) gives $s^2 = 1.65$ for $Y \geqslant 3$. In the light of the above discussion, an asymptotic expansion for F was not considered.

From series (12) and (13), one obtains the temperature distribution

$$\theta = 1 + bF'. \qquad (14)$$

This relation suggests that the temperature is linearly proportional to F', the boundary layer streamwise velocity; this is expected for it is known that the thermal boundary layer thickness must be equal to the velocity boundary layer thickness.

As mentioned earlier, the constant b appearing in equation (14) is to be estimated satisfying the condition that $\theta \to 0$ as $Y \to \infty$. Now, as $Y \to \infty$, $F'(Y) \to s^2$, as discussed earlier, the condition that $\theta(Y) \to 0$ can be satisfied if

$$b = -1/s^2; \quad \text{or} \quad c = -a. \qquad (15)$$

This explicit relation shows that the wall heat transfer rate is directly given by the wall shear stress—a well-established result from the Reynolds analogy. At this stage we can conclude that the series solutions of equations (1)–(4) also lead to an explicit relation between the skin-friction and wall heat transfer.

Further, although relations (6) and (15) are based on the series solutions of equations (1)–(4), relation (15) appears to be simpler than relation (6).

3. CONCLUSION

To sum up, we can say that for the flow under consideration, series solutions also provide an explicit relation, which otherwise follows only from the functional analysis, between the skin-friction and wall heat transfer rate.

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